

# Monte Carlo methods to include the effect of asymmetrical uncertainty sources in wind farm yield assessment

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## Abstract

This article presents a method for incorporating the effect on expected annual energy production of a wind farm caused by asymmetric uncertainty distributions of the applied losses and the nonlinear response in turbine production. The necessity for such a correction is best illustrated by considering the effect of uncertainty in the oncoming wind speed distribution on the production of a wind turbine. Due to the shape of the power curve, variations in wind speed will result in a skewed response in annual energy production. For a site where the mean wind speed is higher than 50% of the rated wind speed of the turbine (in practice all sites with sufficiently high wind speed to motivate the establishment of a wind farm), a reduction in mean wind will cause a larger reduction in annual energy production than a corresponding increase in mean wind would increase the annual energy production. Consequently, the expected annual energy production response when considering the uncertainty of the wind will be lower than the expected annual energy production based on the most probable incoming wind. This difference is due to a statistical bias in the industry standard methods to calculate expected annual energy production of a wind farm, as implemented in tools in common use in the industry. A method based on a general Monte Carlo approach is proposed to calculate and correct for this bias. A sensitivity study shows that the bias due to wind speed uncertainty and nonlinear turbine response will be on the order of 0.5% – 1.5% of expected annual energy production. Furthermore, the effect on expected annual energy production due to asymmetrical distributions of site specific losses, for example, loss of production due to ice, can constitute additional losses of several percent.

## Keywords

Annual energy production, bias wind farm production, uncertainty, wind farm yield, wind resource assessment

## Introduction

Wind resource assessment is a field that is constantly improving as more research, methodology development, new measurement techniques and computational capacity open up new possibilities and increase the quality of wind farm yield estimates. However, experience still shows a systematic over-prediction of average wind farm yield on the order of 5% (Drunic, 2012; Mortensen et al., 2012; Tindal et al., 2007). While there are likely many possible sources for this systematic over-prediction (e.g. not capturing all losses, measurement and modelling errors) as discussed in Clifton et al. (2016), the authors' opinion is that a significant contributor lies in neglecting the inherently asymmetrical nature of uncertainty sources, and asymmetrical response of the wind farm to variations in the oncoming wind conditions.

These effects are typically not accounted for in the current industry standard methods for calculating the expected annual energy production (AEP) of a wind farm (Clifton et al., 2016). In common industry-standard tools (such as WindPRO, WindSim and WindFarmer), the gross energy yield of each wind turbine is calculated based on the expected wind regime at the location of the turbine. Furthermore, it is common to assume that all losses (e.g. availability, wake losses and ice losses) that enter the yield estimation assume their modal value and that any uncertainty is treated as Gaussian uncertainty around an expected P50 value of AEP. In doing so, the distributions of the input parameters and the turbine response

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thereof are not accounted for. The consequence of this is that if either the distribution of the input parameter or the turbine response is asymmetrical, this assumption will lead to AEP estimates that deviate significantly from an estimation where these asymmetries have been taken into account. The intuitive reason for this is that the mean and mode of a probability distribution coincide only if the distribution is symmetric. Consequently, any form of asymmetry induces a bias.

There are different methods available for treatment of the asymmetries in the distribution of the AEP. Lackner et al. (2007) have presented an analytical framework for handling the nonlinear relationship between uncertainties in the wind regime and in AEP uncertainty; however, the method does not consider the bias the asymmetries introduces in the AEP itself.

Due to the nonlinearities, Monte Carlo applications are ideal for quantifying how uncertainties impact the bias and distribution of the AEP. Several studies using Monte Carlo methods are found in the literature; however, the resulting AEP distributions in these studies deviate significantly. Some authors (e.g. Gaß et al., 2011) confirm the misconception that the AEP is generally normally distributed, while others (Hrafnkelsson et al., 2016; Jung et al., 2013) report that the AEP distribution is skewed so that the probabilistic estimate is higher than the deterministic. Others again (Afanasyeva et al., 2016; Lund, 2015; Ucar and Balo, 2009) report that the AEP distribution is skewed towards lower production.

According to Mortensen et al. (2012), a framework for estimation and calculation of uncertainty must be ‘established, disseminated and employed’. Efforts have been made, for example, by a group of companies in the wind industry (DVN KEMA, 2013). The framework presented categorizes losses and uncertainties and mentions briefly that losses (or biases) due to the nonlinear relationship between wind speed and energy should be considered. However, the framework still describes losses and uncertainties as two different aspects. As common methods in use in the industry today do not consider uncertainty when estimating the AEP, we will refer to that as a deterministic estimate. When uncertainties and their distributions are considered in the AEP estimate, we will refer to this as a probabilistic estimate.

As wind turbine technology develops, we see that the capacity factor of new wind farms increase (Weir, 2015). This reduces the difference between the mean wind speed and rated wind speed of the turbines, causing the difference between probabilistic and deterministic estimates to increase. Thus, we believe that it is of utmost importance that methods for calculating the intrinsic bias caused by the nonlinear relationships between wind speed and energy are implemented in the industry standards. This article presents a simple yet rigorous methodology for estimating the probabilistic AEP of a wind farm.

Using the methods proposed in this article, this bias effect is found to be on the order of 0.5%–1.5%. The following sections will introduce the suggested concept and describe the origins of the above-mentioned effects as well as detail the implementation of more robust estimation methods based on Monte Carlo approaches.

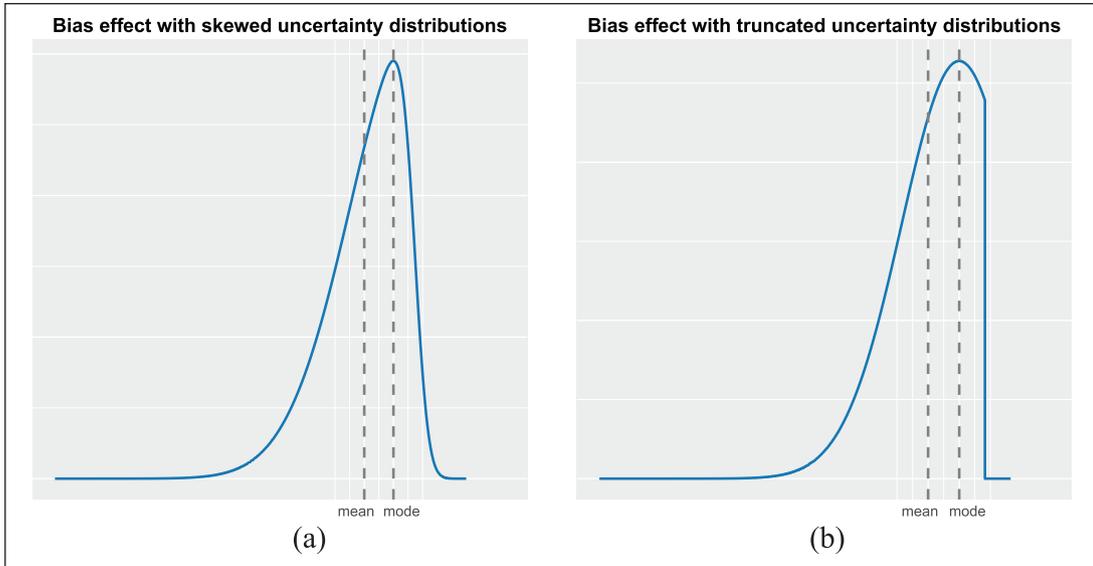
## Description of concept

As outlined in section ‘Introduction’, taking into account asymmetrical sources of uncertainty of the input parameters in AEP estimation can lead to pronounced bias effects when compared to basing the AEP estimation on the modal values of the input parameters alone. In this section, we detail the origin of asymmetrical uncertainty sources and discuss their influence on AEP estimation. Furthermore, we present generic estimation approaches that take these asymmetries into account when estimating AEP.

### *Types of asymmetric uncertainties*

Generally speaking, there are two types of asymmetries in uncertainty distributions.

The first type originates in the nonlinear relation between input and output parameters. The most obvious example is the effect on AEP due to uncertainty in mean wind speed at site. According to DVN KEMA (2013), these uncertainties fall in category 1 through 5 (*Site measurement, Historic Wind Resource, Vertical Extrapolation, Future Wind Variability and Spatial Variation*) and generally contribute to a majority of the total uncertainty (Afanasyeva et al., 2016; Mortensen et al., 2012). The most common assumption is to base the yield assessment directly on the modal value for mean wind speed. However, due to the logistic shape of the power curve, linear variations in mean wind speed at the site will result in a nonlinear response in AEP. At sites where the mean wind speed is higher than 50% of the rated wind speed of the wind turbine (in practice all sites with sufficiently high wind speed to motivate the construction of a wind farm), a reduction in mean wind will cause a larger reduction in AEP than a corresponding increase in mean wind speed would increase the AEP. Therefore, the mean AEP response will be lower than the AEP based solely on the modal value of the mean wind distribution. A similar argument can be made for the relationship between wind speed and wake effects due to the shape of power and thrust curves. However, this effect is found to be an order of magnitude smaller and we disregard it for the remainder of this article.



**Figure 1.** Exemplary illustration of bias effects due to asymmetric distributions: (a) skewed normal distribution and (b) truncated normal distribution.

The second type originates in skewed and/or truncated loss distributions. As a consequence, extreme values on one tail are receiving larger probability mass and are thus more likely to occur. Such symmetric uncertainty distributions arise whenever there is a physical (or practical) limit in either direction of the input domain. This can be, for instance, ice loss where the loss distribution is truncated to the right at 0; or production-based turbine availability in the warranty period where the uncertainty distribution can be truncated to the left at the warranty level if the risk is mitigated contractually. An exemplary illustration of how these asymmetries introduce a bias is given in Figure 1 where the induced (relative) bias is defined as the ratio between mean and mode of the asymmetric uncertainty distribution.

### Bias calculation with Monte Carlo methods

In this section, we present a general approach based on Monte Carlo methods to take the two types of asymmetries into account when estimating AEP. In both cases, we base our approach on the fact that an integral with respect to some suitably defined probability distribution  $F$  can be accurately approximated by a sample average of random variables drawn from the distribution of interest. As we are interested in the mean of some distribution  $F$  over its domain  $\Omega$  we approximate the integral

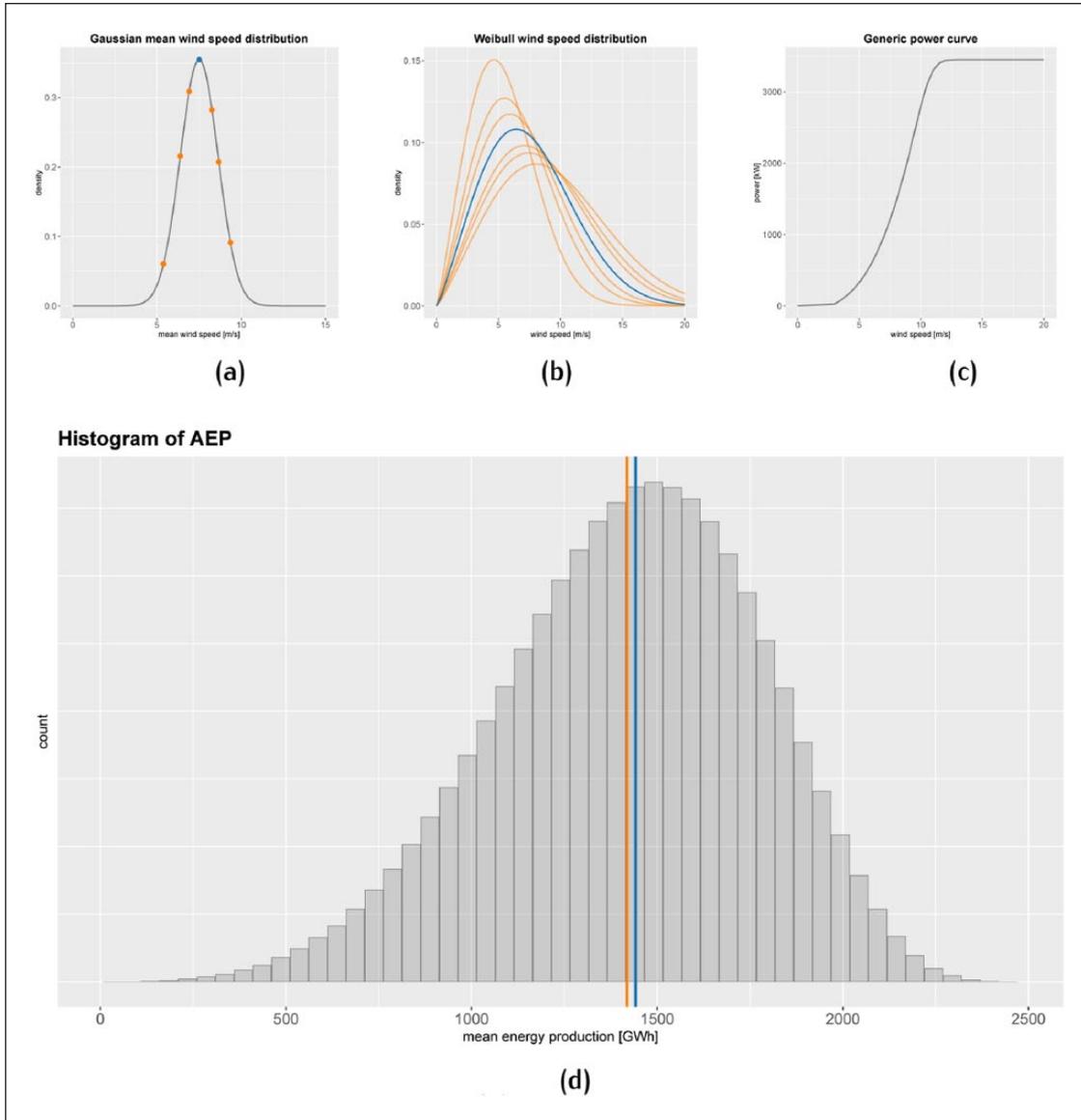
$$\mu_x = \int_{\Omega} x \, dF(x)$$

with the Monte Carlo sum

$$\hat{\mu}_x = \frac{1}{N} \sum_{i=1}^N \xi_i$$

where  $(\xi_i)_{i=1}^N$  is now an identically and independently drawn sample of size  $N$  from  $F$ . By the law of large numbers  $\hat{\mu}_x$  converges to  $\mu_x$  as  $N \rightarrow \infty$ .

A Monte Carlo approach has many desirable characteristics in the problem setting considered here. Most prominently, when calculating the bias which is due to nonlinear turbine response functions, the advantage is that a precise mathematical formulation of the asymmetry considered is not required and that it can be defined implicitly. Similarly, when dealing with asymmetries due to skewed and/or truncated AEP loss distributions, a Monte Carlo approach allows for a very general specification of the uncertainty distribution which would otherwise be impossible to integrate in closed form. All that is required is that one may generate random draws from the distribution of interest.



**Figure 2.** Illustration of Monte Carlo steps in AEP estimation that are due to nonlinear turbine response functions. The blue vertical line in panel (d) indicates the estimate  $\text{AEP}^*$  based on the modal wind speed (blue dot in panel (a)), whereas the orange vertical line in panel (d) indicates the Monte Carlo estimate  $\text{AEP}$ . (a) Gaussian prior, (b) Weibull distribution, (c) power curve and (d) AEP bias.

## Implementation and discussion

In this section, we detail the implementation of each step of our suggested Monte Carlo estimation approaches and discuss the sensitivity of the bias estimates with respect to variations in the key input parameters.

### Implementation

*Calculating bias effects due to nonlinear response.* In the case of asymmetric uncertainty distributions due to nonlinear turbine response functions, we suggest to calculate the bias via the following Monte Carlo algorithm:

1. Define the distribution of the mean wind speed at site. For the remainder of the article, we assume a Gaussian distribution for mean wind speeds with mean  $\mu_{\bar{w}}$  and standard deviation  $\sigma_{\bar{w}}$  (see panel (a) of Figure 2).

2. Generate an independently and identically distributed sample of size  $N$  of mean wind speeds  $(\bar{w}_i)_{i=1}^N$  from a Gaussian distribution with parameters  $\mu_{\bar{w}}$  and  $\sigma_{\bar{w}}$ , that is,  $\bar{w}_i \sim \mathcal{N}(\mu_{\bar{w}}, \sigma_{\bar{w}}^2)$  for all  $i = 1, \dots, N$ . A sensitivity study showed that taking  $N = 2 \times 10^6$  is sufficient to yield convergence of the Monte Carlo algorithm (see the orange dots in panel (a) of Figure 2).
3. For each  $i = 1, \dots, N$ , define a Weibull distribution of wind speeds with shape parameter  $k_i$  and scale parameter  $\lambda_i$  (see the orange curves in panel (b) of Figure 2). For the remainder of the article, we assume a constant value for the shape parameter, that is,  $k_i = k$  for all  $i = 1, \dots, N$ . The scale parameter is then taken such that the mean of the Weibull distribution matches the corresponding mean wind speed generated in step 2, that is

$$\lambda_i = \frac{\bar{w}_i}{\Gamma\left(1 + \frac{1}{k}\right)}, \quad \text{for all } i = 1, \dots, N$$

where  $\Gamma$  denotes the Gamma function.

4. For each  $i = 1, \dots, N$ , calculate an AEP estimate  $AEP_i$  by translating the Weibull distribution for wind speeds  $w_i$  through the warranted power curve  $P(w)$  (see panel (c) in Figure 2).
5. Denote by  $\overline{AEP} = (1/N) \sum_{i=1}^N AEP_i$  the mean AEP estimate resulting from Monte Carlo simulations and by  $AEP^*$  the AEP estimate based on the mode of the mean wind speed distribution (see the blue features in Figure 2). The relative bias is then calculated as

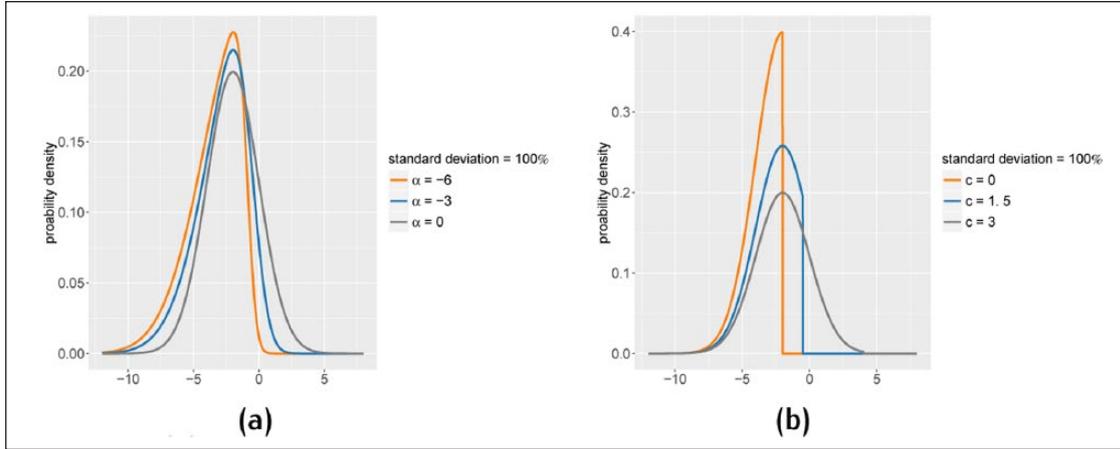
$$\text{Bias} = \frac{\overline{AEP}}{AEP^*} - 1$$

Panel (d) of Figure 2 shows the bias effect as calculated by the above Monte Carlo algorithm for an onshore wind farm with measured mean windspeed of  $\mu_{\bar{w}} = 7.5$  m/s and standard deviation of  $\sigma_{\bar{w}} = 8\%$ . The shape parameter  $k$  was found by fitting a Weibull distribution to the measured wind speeds at site and takes on the value  $k = 2.2$ . Based on the Monte Carlo algorithm described above, the AEP estimate  $\overline{AEP}$  is approximately 1.6% lower than the estimate  $AEP^*$  which is based on the modal mean wind speed alone. As such, incorporating asymmetries in the uncertainty distribution can account for a large part of historical overpredictions in AEP estimation.

*Calculating bias effects due to skewed and/or truncated loss distributions.* The main challenge when calculating bias effects that are due to skewed and/or truncated loss distributions is how to define these distributions in mathematically precise terms. In most real-world applications, loss distributions are assumed to be Gaussian and identified with their location and scale parameters  $\mu$  and  $\sigma$ , respectively. While truncating a normal distribution is straightforward, an elegant way to incorporate skewness in a normal distribution while retaining its favourable properties has been discussed in the monograph of Azzalini (2013). Whereas a closed-form expression for the mean is available, this is not the case for the mode or median of the skew normal distribution. Similarly, more general distributions such as the recently studied family of lognormal-normal mixture distributions do not exhibit a closed-form solution for the mean (see Yang, 2008).

The Monte Carlo approach we suggest is robust against such distributional complexities and allows us to estimate the mean and mode as long as one can generate a sample of random variables from the distribution of interest. In what follows, we illustrate our approach with skew and truncated normal distributions to simplify our exposition and note that more general distributions can be easily incorporated:

1. (a) Consider a skewed normal distribution with probability density function  $f_{sn}(x; \theta, \sigma^2, \alpha)$  with location parameter  $\theta$ , scale parameter  $\sigma$  and shape parameter  $\alpha$ . Note that the parameters  $\theta$  and  $\sigma$  attain their usual interpretation of mean and standard deviation only in the special case where  $\alpha = 0$ , in which case the density  $f_{sn}(x; \theta, \sigma^2, \alpha)$  reduces to the density of a normal distribution with mean  $\theta$  and standard deviation  $\sigma$ . The shape parameter  $\alpha$  governs the degree of skewness or asymmetry. For illustration, several skew normal distributions are plotted in panel (a) of Figure 3 that show the effect of varying the shape parameter  $\alpha$  while holding all other parameters constant.
- (b) Consider a (right) truncated normal distribution with probability density function  $f_{tn}(x; \mu, \sigma^2, c)$  where  $\mu$  and  $\sigma$  are the usual location and scale parameters and  $c$  denotes the (upper) bound. For illustration, several distributions are plotted in panel (b) of Figure 3 that show the effect of varying the cut-off parameter  $c$  while holding all other parameters constant. Note that each distribution is rescaled such that its integral over its support equals one.



**Figure 3.** Illustration of varying shape parameters: (a) skewed normal distribution and (b) truncated normal distribution.

2. Generate a large sample of size  $N$  of random variables  $(\xi_i)_{i=1}^N$  from either the skew normal distribution with density  $f_{sn}$  or from the truncated normal distribution with density  $f_{tn}$ . Order the sample such that  $\xi_1 \leq \xi_2 \leq \dots \leq \xi_N$ .
3. Estimate the mean by

$$\bar{M} = \frac{1}{N} \sum_{i=1}^N \xi_i$$

4. Several estimators for the mode are available (see Bickel, 2006 and the references therein). One estimator that is directly computable is due to Grenander (1965) and takes the form

$$M_{p,k}^* = \frac{\frac{1}{2} \sum_{i=1}^{N-k} (\xi_{i+k} + \xi_i) / (\xi_{i+k} + \xi_i)^p}{\sum_{i=1}^{N-k} 1 / (\xi_{i+k} + \xi_i)^p} \quad \text{where one takes } 1 < p < N \text{ and } k > 2p \text{ to ensure consistency of the}$$

estimator.

5. The (relative) bias is then calculated as

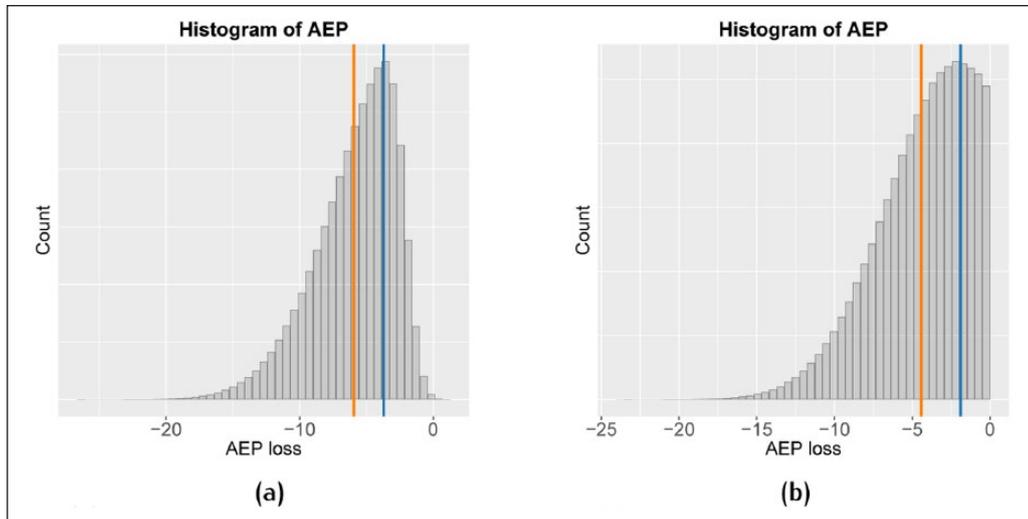
$$\text{Bias} = \frac{\bar{M}}{M_{p,k}^*} - 1$$

Figure 4 shows the skewed (left panel) and truncated (right panel) AEP loss distributions based on a sample of  $N = 2 \times 10^6$  random draws from  $f_{sn}$  and  $f_{tn}$ , respectively. The mean estimate is depicted in orange, while the mode is depicted in blue.

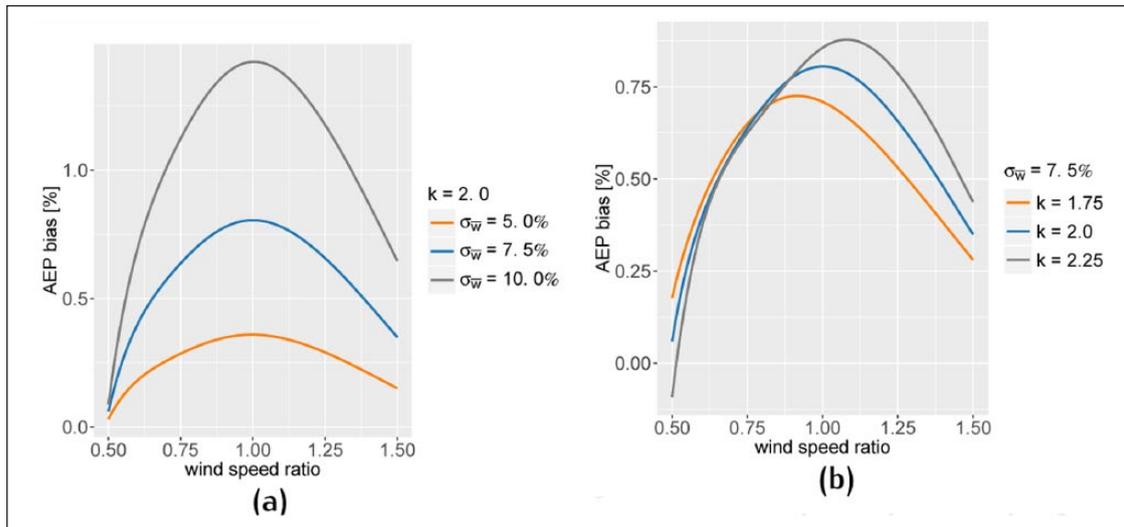
## Discussion

In this section, we discuss the effect that different parameters have on the magnitude of AEP bias estimates.

Consider first the case of nonlinear turbine response functions. The two panels in Figure 5 show how the AEP bias varies with the wind speed ratio (i.e. the ratio of mean to rated wind speed). In panel (a) of Figure 5, the shape factor for the Weibull distributions of wind speeds is kept fixed to show the effect different standard deviations of mean wind speeds have on AEP bias. We note first that the higher the standard deviation, the higher the AEP bias, irrespective of the wind speed ratio. Second, the AEP bias attains a global maximum for a certain mean to rated wind speed ratio. While in panel (a) this maximum is attained for a wind speed ratio of 1, we see in panel (b) that the location of this maximum actually depends on the shape factor  $k$  of the Weibull distribution of wind speeds.



**Figure 4.** Illustration of bias effects: (a) skewed normal distribution and (b) truncated normal distribution.

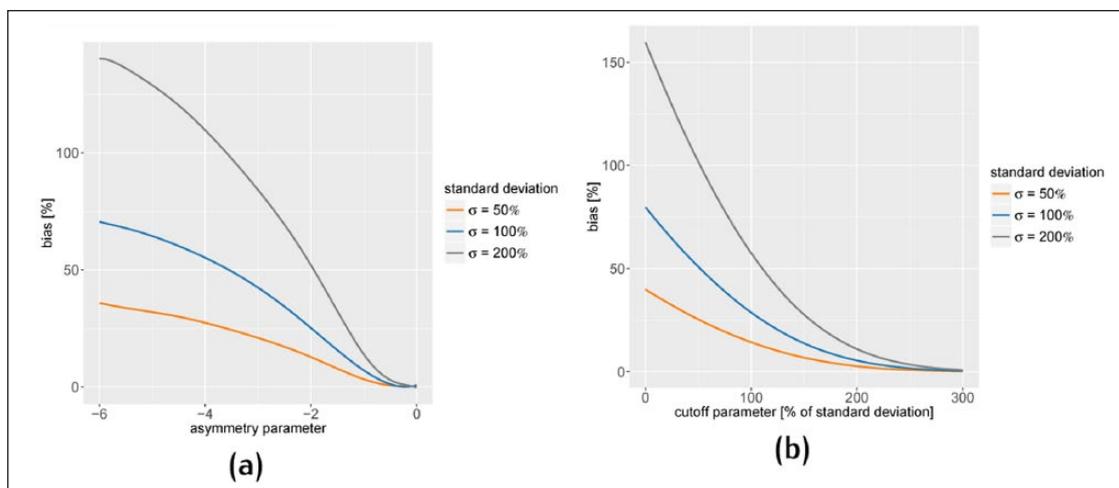


**Figure 5.** Sensitivity analysis of bias effects: (a) sensitivity analysis: varying  $\sigma_w$  and (b) sensitivity analysis: varying  $k$ .

Consider next the case of skewed or truncated uncertainty distributions. To facilitate the interpretation of the sensitivity analysis, we consider three different scenarios defined by the relative standard deviation relative to its mean. In the case of the skew normal distribution, the amount of asymmetry imposed is increasing with the (absolute) value of the parameter  $\alpha$  and perfect symmetry is attained for  $\alpha = 0$ . Negative values of  $\alpha$  impose negative skew and vice versa. Panel (a) in Figure 6 shows the effect of varying  $\alpha$  from high (absolute) values to 0. Naturally, the higher the asymmetry, the higher the induced difference between the mean and the mode of the distribution and thus the higher bias. Also, as the amount of asymmetry is approaching 0, so is the induced bias. Panel (b) in Figure 6 shows the effect of varying the cut-off parameter  $k$  of a truncated normal distribution. Truncation starts at the mode of a symmetric normal distribution ( $c = 0$ ) and moves to the right end of the distribution by adding fractions of the relative standard deviation to the mean. Naturally, the further the cut-off moves to the right (in the right-truncated case), the smaller the distance between the mean and the mode, and hence, the bias will be. As before, the bias effects are more pronounced, the larger the relative standard deviation is.

### Conclusion

A method for incorporating the effect of asymmetric uncertainty distributions and nonlinear turbine response in wind turbine yield assessments has been presented. It has been shown that the bias introduced by the current standard method for



**Figure 6.** Sensitivity analysis of bias effects: (a) sensitivity analysis: varying asymmetry parameter  $\alpha$  and (b) sensitivity analysis: varying cut-off parameter  $c$ .

estimating the expected AEP, where the expected mean AEP is calculated based on the modal value of the input parameters and the Gaussian uncertainty about the mean is assumed, is on the order of 0.5%–1.5% due to wind speed uncertainty and nonlinear turbine response. Furthermore, the effect on expected AEP due to asymmetrical distributions of site specific losses, for example, loss of production due to ice, can constitute additional losses of several percent. Consequently, this bias is considered a significant cause of the systematic over-predictions evident in pre-construction estimates of wind farm yield.

### Declaration of conflicting interests

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